

Chapter 14 — Proportional Relationships in Tables, Graphs, and Equations — Teacher Guide

Chapter at a glance

Lesson duration	5 lessons, 45–65 minutes each (see pacing table below)
Materials	Input/output cards, unit-rate strips, coordinate grids, graph paper, equation cards, tables with missing entries, origin stickers, proportional/non-proportional relationship cards, the three chapter diagrams
Launch question	“A shop charges \$5 per notebook. Show this as a context, a table, a graph, and an equation — what stays the same across all four?”
Key representation	The table-graph-equation translation map — keep it visible throughout instruction
Anticipated responses	Correct: checks the constant quotient across all rows and confirms the graph passes through the origin. Partial: computes correctly but can’t explain why the origin matters. Incorrect: calls any rising table or straight line proportional without checking the quotient or intercept.
If the readiness check fails	Reteach unit rates and proportions from Chapters 11–13 before starting the Launch; do not skip the quantity-pair card step
Talk prompt	“Does the output divided by input stay constant? What happens at zero input?”
Exit ticket	5 items — check a table, write an equation, evaluate the equation, test a graph’s intercept, explain a non-proportional case (see Section 14)
Reteach / extend	Reteach: whole-number constants only, with physical quantity-pair cards. Extend: fractional and decimal constants, negative examples, matching-rate/different-intercept comparisons.

Purpose and pacing

Chapter 14 translates proportional reasoning into the language of variables and coordinate graphs. The central idea is not symbol manipulation. It is that one constant multiplicative relationship survives every representation.

Lesson	Focus	Suggested time
Lesson 1	Readiness, launch, context, and quantity-pair cards	45–60 minutes
Lesson 2	Tables, constant quotient, and equations	50–65 minutes
Lesson 3	Graphs, origin, and table–graph–equation translation	50–65 minutes
Lesson 4	Non-proportional relationships, misconceptions, and Tripwire	45–60 minutes
Lesson 5	Mixed practice, Exit Ticket, and targeted reteaching	45–60 minutes

Materials

Prepare input/output cards, unit-rate strips, coordinate grids, graph paper, equation cards, tables with missing entries, origin stickers, and proportional/non-proportional relationship cards.

Teacher launch guidance

Use the notebook context to build the four representations in sequence. Keep the same quantities and units visible while the form changes. Ask students to explain what the 5 means in the table, graph, and equation.

When introducing the graph, do not present the origin as a visual rule only. Ask students what zero input means in the context. A direct proportional relationship must produce zero output when the input is zero.

Use the taxi example to distinguish a constant rate from a direct proportion. A relationship can increase at a constant amount and still have a starting value that makes it non-proportional.

Teacher prompts

Purpose	Prompt
Identify variables	“What is the input? What depends on it?”
Find k	“What is output divided by input?”
Interpret k	“What does k mean per one unit?”
Check table	“Does the quotient stay constant?”
Check graph	“Does the line include the origin?”
Check equation	“Is there an added starting amount?”

Purpose	Prompt
Translate	"How does the same relationship appear in the other form?"
Diagnose	"What evidence says proportional or not?"

Differentiation

Students needing concrete support

Use quantity-pair cards and build tables from physical groups. Put an origin card beside the "zero input" situation. Delay formal equation writing until students can state the unit rate in words.

Students needing language support

Use the frames:

- "The input is _____ and the output is _____."
- "The constant of proportionality is _____ because _____ $\div$ _____ = _____."
- "The equation is _____."
- "The graph is/is not proportional because _____."
- "At zero input, the output should be _____."

Students ready to extend

Use fractional constants, negative examples, tables with missing values, graph interpretation, and models with starting amounts. Ask students to write two equations that have the same rate but different intercepts and explain the difference.

Formative assessment

Record whether students identify variables, compute constant quotients, interpret k with units, include zero, recognize the origin, write $y = kx$, distinguish a starting amount, and translate across representations. A correct graph without a labeled relationship should be marked developing.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Find k in $y = kx$ when $(x,y) = (4,12)$. Identify whether a graph through $(0,0)$ is enough evidence by itself. Complete a table for $y = 2.5x$. Explain what the slope means in context.

Strategy Studio guidance: Expected ideas: $y = 2.5x$ and $k = 2.5$ units per x -unit; a nonzero intercept breaks direct proportionality; $y = 3x$ gives $(7,21)$; $y = 2.5x + 4$ includes a fixed amount. Ask students to state units for slope and intercept.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error.

Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: y -values are 0, 5, 10, 15. Altered: $y = 2.5x$ is direct; $y = 2.5x + 4$ has a nonzero intercept. Non-routine: the same rate can describe a proportional relationship or a fixed-fee relationship. Repair: linearity alone is not enough; direct proportionality requires $y = kx$ and passage through the origin.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

Ready-to-Learn Check

Amount for one unit.

\$12.

15.

(0,0).

28.

Constant of proportionality or unit rate.

No; it includes a starting amount.

Worksheet A

- Yes.** Likely wrong answer: says “no” or “yes” based on the rising pattern alone, without computing $y \div x$ for each pair. Misconception: assuming any increasing table is automatically proportional (or non-proportional) without checking the quotient. Next move: “compute $y \div x$ for every pair — is it the same number each time?”
- $k = 3$.** Common flaw: identifies the pattern of “adding 3 each time” instead of the multiplicative constant. Next move: “is 3 the amount added, or the amount multiplied?”
- $y = 3x$.** Likely wrong answer: $y = x + 3$ (writes an additive rule instead of a multiplicative one). Misconception: “addition is enough” — mistaking a constant difference for the proportional relationship. Next move: “does y equal x plus 3, or x times 3? Test with $x = 2$.”
- No; the quotients are 4, 3, and 2.67.** Likely wrong answer: “yes,” because the output is increasing steadily. Misconception: assuming any rising table is proportional. Next move: “compute $y \div x$ for each pair — do they match?”
- $C = 2p$.** Likely wrong answer: $C = p + 2$ (additive rule instead of multiplicative). Misconception: same as #3. Next move: “does the cost add 2, or multiply by 2, for each pencil?”
- \$18.** Likely wrong answer: \$11 (adds 9 and 2 instead of multiplying). Misconception: confusing scaling with combining. Next move: “what does one pencil cost? Multiply by 9.”
- $d = 60t$.** Likely wrong answer: $d = t + 60$ (additive rule). Misconception: same as #3. Next move: “does distance add 60, or multiply by 60, for each hour?”
- 240 km.** Likely wrong answer: 64 km (adds 60 and 4 instead of multiplying). Misconception: same as #6. Next move: “what is the speed? Multiply by 4 hours.”
- Yes, it includes (0,0).** Common flaw: says “yes” without naming the origin specifically as the required point. Next

- move: ask what point must be on the graph for a direct proportion.
5. **At zero input, the output is 4 rather than 0.** Likely wrong answer: says "it's not proportional" without identifying the specific zero-input evidence. Misconception: ignoring context — not checking what happens at the origin. Next move: "what is y when $x = 0$? Should it be 0 for a direct proportion?"

Worksheet B

- Yes, because $y/x = 5$.** Likely wrong answer: says "yes" based on the rising pattern alone, without computing the quotient. Misconception: assuming any increasing table is proportional. Next move: "compute $y \div x$ for every pair."
- No; quotients are 6, 4.5, and 4.** Likely wrong answer: "yes," because the output increases by a steady amount (3 each time). Misconception: mistaking a constant difference for proportionality. Next move: "compute $y \div x$ for each pair — do they match?"
- $k = 5$, so $y = 5x$.** Common flaw: identifies the additive pattern (adding 5 each row) instead of the multiplicative constant. Next move: "is 5 the amount added, or the amount multiplied?"
- $P = 22h$.** Likely wrong answer: $P = h + 22$ (additive rule instead of multiplicative). Misconception: "addition is enough." Next move: "does pay add 22, or multiply by 22, for each hour?"
- \$176.** Likely wrong answer: \$30 (adds 22 and 8 instead of multiplying). Misconception: confusing scaling with combining. Next move: "what is the hourly rate? Multiply by 8 hours."
- 332.5 km.** Likely wrong answer: 98.5 km (adds 95 and 3.5 instead of multiplying). Misconception: same as #5. Next move: "what does the equation say to do with t ?"
- Starting amount of 2.** Likely wrong answer: says "it's not proportional" without naming the specific starting-amount evidence. Misconception: ignoring context. Next move: "what is y when $x = 0$?"
- Answers vary; proportional must be $y = kx$, non-proportional may include $+b$.** Common flaw: writes two equations that are both proportional, or both non-proportional, without contrast. Next move: check that exactly one equation has an added constant.
- Straight line through the origin with rate 4.** Common flaw: describes the line as "steep" or "increasing" without naming that it passes through the origin. Next move: ask what point must be included for a direct proportion.
- 30.** Likely wrong answer: 14.5 (adds 2.5 and 12 instead of multiplying). Misconception: same as #5. Next move: "does $y = 2.5x$ mean add or multiply?"

Worksheet C

- 0:\$0, 1:\$3, 2:\$6, 3:\$9, 4:\$12.** Likely wrong answer: starts the table at 1 notebook instead of 0, omitting the zero pair. Misconception: omitting zero — not testing the origin case. Next move: "what should the cost be for zero notebooks?"
- $C = 3n$.** Likely wrong answer: $C = n + 3$ (additive rule instead of multiplicative). Misconception: "addition is enough." Next move: "does cost add 3, or multiply by 3, for each notebook?"
- 0:\$5, 1:\$7, 2:\$9, 3:\$11, 4:\$13.** Common flaw: computes the pattern correctly but doesn't include the zero row, missing the key evidence for the next question. Next move: require the zero-input row in every table.

4. **No; the output at zero is \$5.** Likely wrong answer: “yes,” because the table increases by a steady \$2 each row. Misconception: ignoring context — mistaking a constant difference for proportionality. Next move: “what is the cost at zero kilometres? Should it be \$0 for a direct proportion?”
5. **$d = 45t$; 292.5 km.** Likely wrong answer: 51.5 km (adds 45 and 6.5 instead of multiplying). Misconception: confusing scaling with combining. Next move: “what is the speed? Multiply by 6.5 hours.”
6. **$k = 4$, $y = 4x$.** Common flaw: identifies the additive pattern (adding 12 each row) instead of the multiplicative constant found by dividing. Next move: “compute $y \div x$ for each pair — what constant do you get?”
7. **No; quotients are 10, 7.5, and 6.67.** Likely wrong answer: “yes,” because the output increases steadily by 5. Misconception: assuming any rising table is proportional. Next move: “compute $y \div x$ for each pair — do they match?”
8. **Answers vary but must include a constant multiplier.** Common flaw: creates a context where the amount added is constant but the multiplier is not, describing it as proportional anyway. Next move: check that $y \div x$ is constant for the created example.
9. **It is a line that does not pass through the origin.** Common flaw: says the graph is “just a line” without stating specifically that it misses the origin. Next move: ask where the line crosses the vertical axis.
10. **They should show the same input, output, and constant rate.** Common flaw: says the three representations “should match” without naming what specifically must agree (the constant k). Next move: ask the student to identify k in all three forms of one example.
6. **$C = 250n$; 6 bikes cost \$1,500. The 3-month tracking period and the number of employees (5) were not needed to answer the question.** Likely wrong answer: works the 3 months or 5 employees into the calculation somehow. Misconception: assuming every number in a word problem must be used. Next move: “what does the question actually ask for? Which numbers answer that?”
7. **Answers vary; must describe a situation where 12 is a constant rate (such as dollars per item or km per hour), and explain that zero input must produce zero output for the relationship to be proportional.** Common flaw: gives a context with the right constant but doesn’t explain why the origin makes sense in that context. Next move: ask “what would zero items or zero hours mean here? Should the output be zero?”

Extend, Tripwire, and Exit Ticket

$y = 0.75x$; k means 0.75 output units per input unit. 2–3. Answers vary but must be checked with y/x . 4. It has a nonzero intercept. 5. $k = 4.5$, so $y = 4.5x$. 6. Answers vary but must identify zero behavior.

Tripwire: not proportional because the quotient changes.

Exit Ticket:

1. yes, $k = 4$;
2. $y = 4x$;
3. 22.5 cm.;
4. no, because it does not pass through the origin;
5. the +3 is a starting amount.

